General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MFP1

Unit Further Pure 1

Friday 18 January 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A curve passes through the point (1, 3) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{x}{1 + x^3}$$

Starting at the point (1, 3), use a step-by-step method with a step length of 0.1 to estimate the value of y at $x = 1.2$. Give your answer to four decimal places.
(5 marks)

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2 (a) Solve the equation $w^2 + 6w + 34 = 0$, giving your answers in the form $p + q\mathrm{i}$, where p and q are integers. *(3 marks)*

(b) It is given that $z = \mathrm{i}(1 + \mathrm{i})(2 + \mathrm{i})$.

(i) Express z in the form $a + b\mathrm{i}$, where a and b are integers. *(3 marks)*

(ii) Find integers m and n such that $z + mz^* = n\mathrm{i}$. *(3 marks)*

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3 (a) Find the general solution of the equation

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

(b) Use your general solution to find the exact value of the greatest solution of this equation which is less than 6π. *(2 marks)*

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4 Show that the improper integral $\displaystyle\int_{25}^{\infty} \frac{1}{x\sqrt{x}}\,dx$ has a finite value and find that value.
(4 marks)

5 The roots of the quadratic equation

$$x^2 + 2x - 5 = 0$$

are α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Calculate the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $\alpha^3\beta + 1$ and $\alpha\beta^3 + 1$. *(5 marks)*

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6 (a) The matrix $\mathbf{X}$ is defined by $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$.

(i) Given that $\mathbf{X}^2 = \begin{bmatrix} m & 2 \\ 3 & 6 \end{bmatrix}$, find the value of m. *(1 mark)*

(ii) Show that $\mathbf{X}^3 - 7\mathbf{X} = n\mathbf{I}$, where n is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(4 marks)*

(b) It is given that $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(i) Describe the geometrical transformation represented by $\mathbf{A}$. *(1 mark)*

(ii) The matrix $\mathbf{B}$ represents an anticlockwise rotation through $45°$ about the origin.

Show that $\mathbf{B} = k\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, where k is a surd. *(2 marks)*

(iii) Find the image of the point $P(-1, 2)$ under an anticlockwise rotation through $45°$ about the origin, followed by the transformation represented by $\mathbf{A}$. *(4 marks)*

Turn over ▶

7 The variables y and x are related by an equation of the form

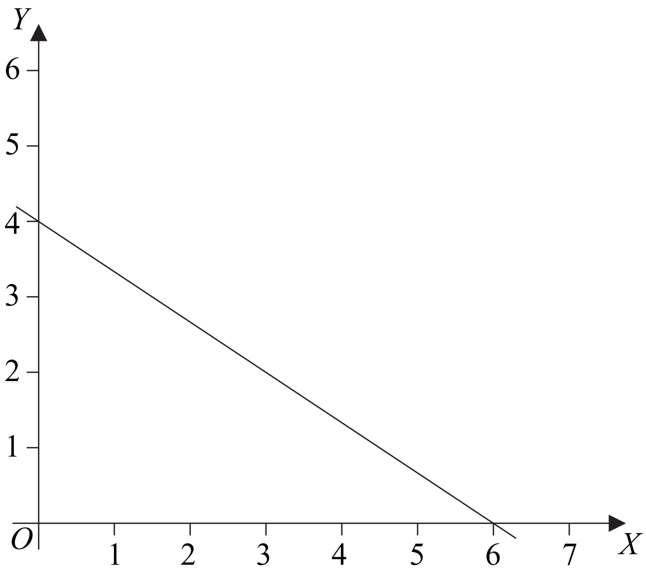
$$y = ax^n$$

where a and n are constants.

Let $Y = \log_{10} y$ and $X = \log_{10} x$.

(a) Show that there is a linear relationship between Y and X. *(3 marks)*

(b) The graph of Y against X is shown in the diagram.



Find the value of n and the value of a. *(4 marks)*

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8 (a) Show that

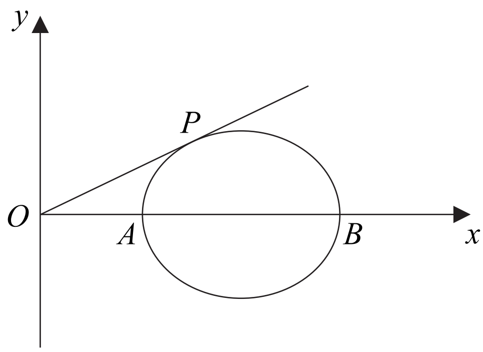
$$\sum_{r=1}^{n} 2r(2r^2 - 3r - 1) = n(n+p)(n+q)^2$$

where p and q are integers to be found. *(6 marks)*

(b) Hence find the value of

$$\sum_{r=11}^{20} 2r(2r^2 - 3r - 1)$$ *(2 marks)*

9 An ellipse is shown below.



The ellipse intersects the x-axis at the points A and B. The equation of the ellipse is

$$\frac{(x-4)^2}{4}+y^2=1$$

(a) Find the x-coordinates of A and B. *(2 marks)*

(b) The line $y = mx$ $(m > 0)$ is a tangent to the ellipse, with point of contact P.

(i) Show that the x-coordinate of P satisfies the equation

$$(1+4m^2)x^2 - 8x + 12 = 0$$ *(3 marks)*

(ii) Hence find the exact value of m. *(4 marks)*

(iii) Find the coordinates of P. *(4 marks)*